Complexity

steps

$f \in O(g)$

$n \to \infty$

$\exists c > 0 \; \underbrace{\exists n_0 > 0 \; \forall n > n_0}_{n \text{ large enough}} \; |f(n)| \le c\,|g(n)|$

upto a constant.

$10n + 100 \in O(n)$

$3n + 4 \in O(n)$

$\exists_c \boxed{\exists_{n_0} \forall_{n>n_0}} \; 3n+4 \le C \cdot n$

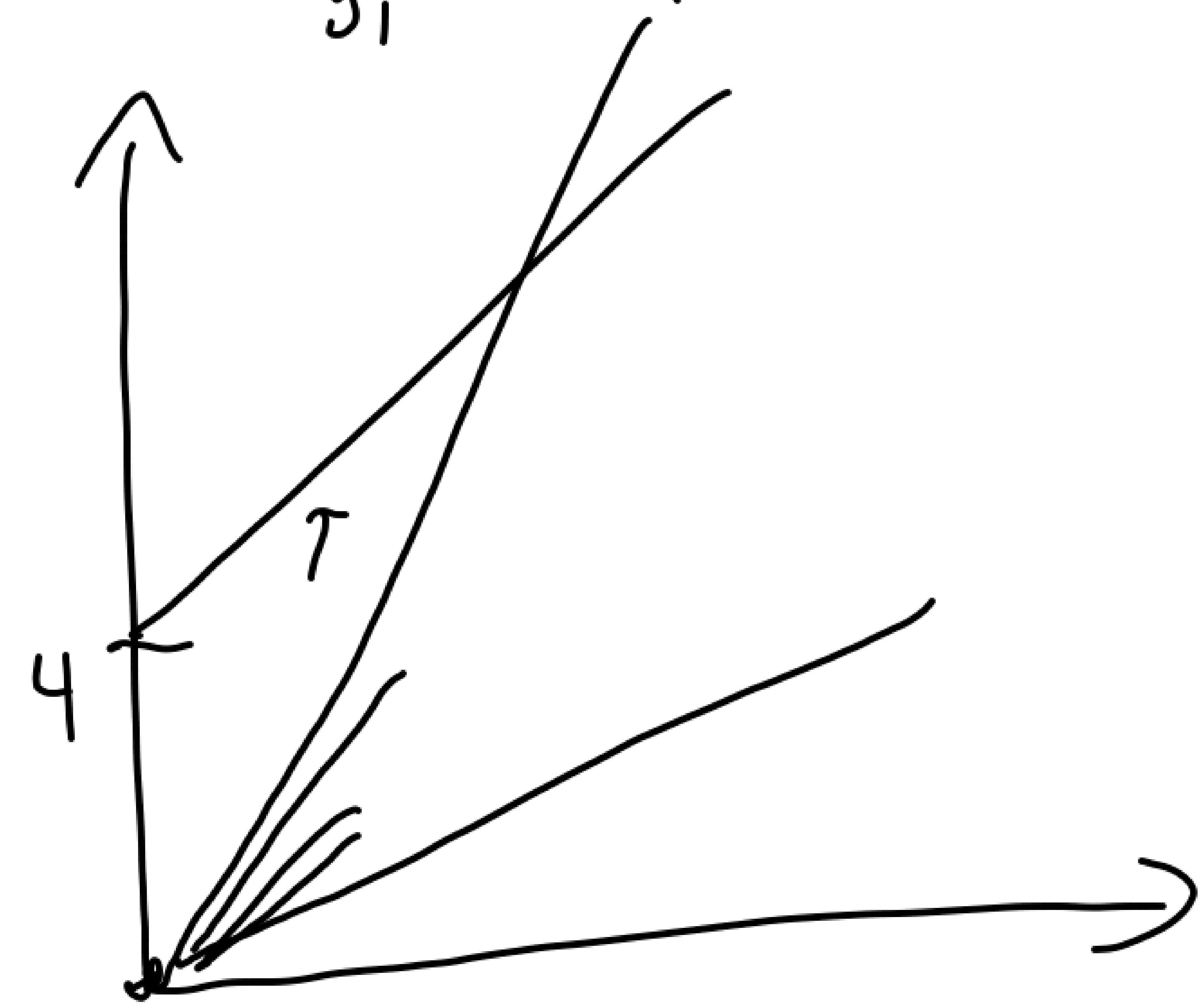
$C = 2 \cdot 3 = 6$

$$3n+4 \le 6n$$

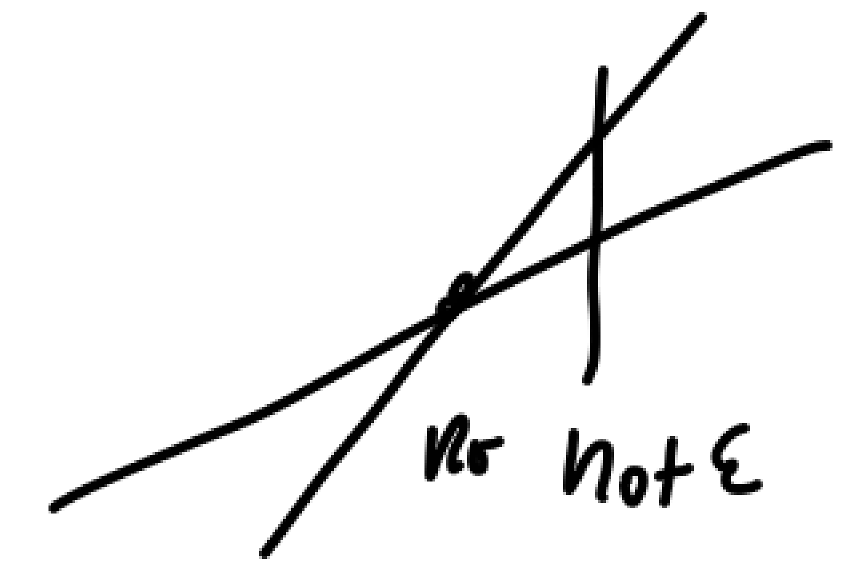
```
a;                12
b;
for i=1...e
    c;            n·3
    d;
    e;
f;
g;
```

n

12



$$3n + 4 \le 6n \quad \vert -3n \qquad \text{What is } n_0$$



$$4 \le 3n$$

$$n \ge \frac{4}{3}$$

$n_0 = \frac{4}{3}$ or 2 if you want integer n_0

$C = 6,\ n_0 = 2$

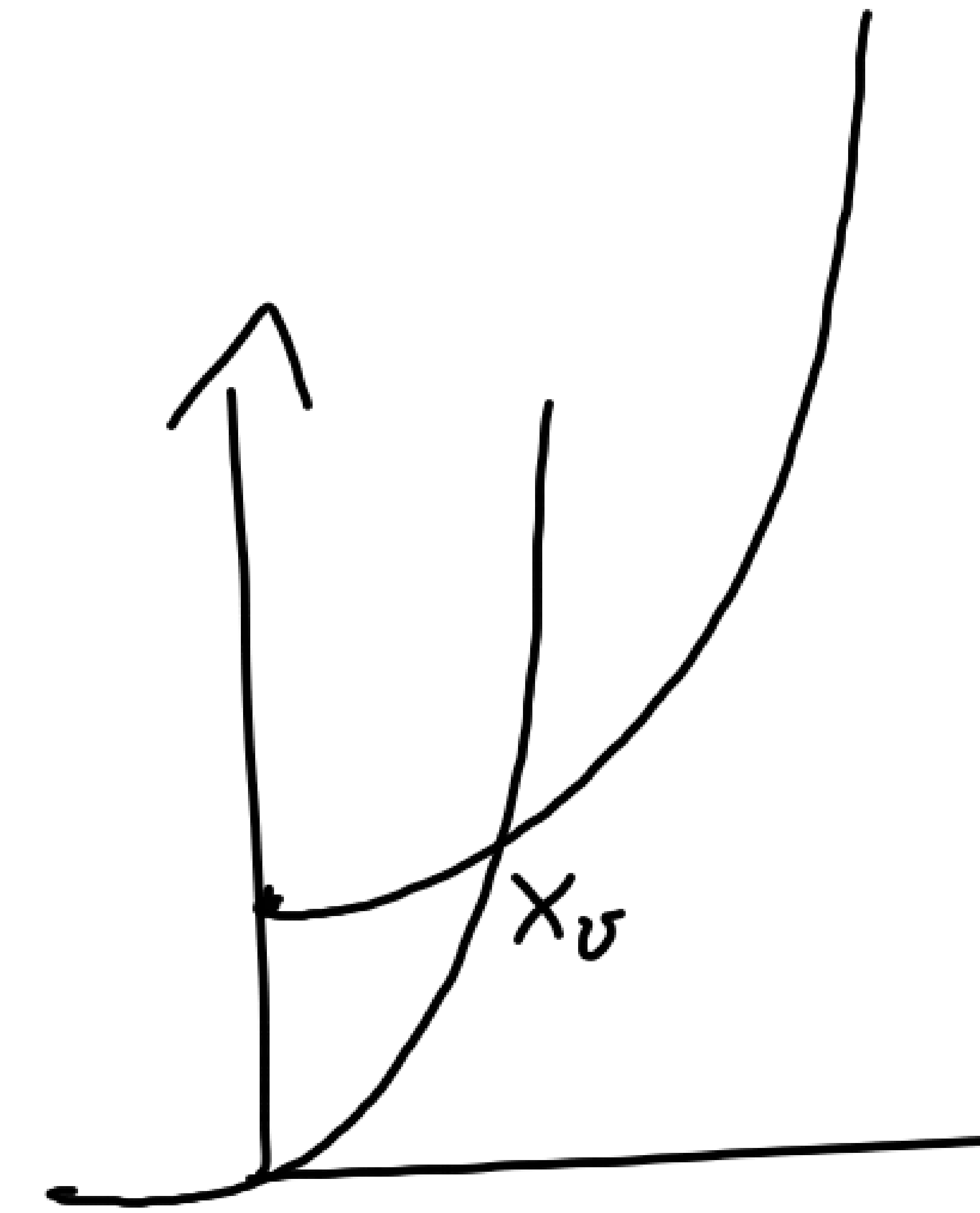
Shown $3n+4 \in O(n)$

$a, b, q > 0$

$ax^2 + bx + q \in O(x^2)$

$\exists_c \exists_{n_0} \forall_{n > n_0}$ $ax^2 + bx + q \leq Cx^2$

$C = 2a + 2b = 2(a+b)$



$\cancel{ax^2} + bx + q \leq \cancel{2} ax^2 + 2bx^2$

$bx + q \leq \underbrace{\cancel{ax^2}}_{>0} + 2bx^2$? $x^2 > x$?

$\leq 2bx \leq 2bx^2$

$bx + q \leq$!

$\rightsquigarrow$ Homework

There is a $()^2$ fct. that intersects

$$\mathcal{O}(cf(n)) = \mathcal{O}(f(n))$$

Composition (Steps run after each other)

$$\mathcal{O}(A(n) + B(n)) = \mathcal{O}(\max A(n), B(n))$$

for $i = 0 \dots 3$
$x_i = \dots$
$\mathcal{O}(1)$

For loops:

a) the loop has a constant number of iterations

for $i = 1 \dots n$ $\mathcal{O}(n$
~

b) " ——— linear ———

for $i = 1 \dots n$
for $j = 1 \dots n$ $\mathcal{O}(n$
$a_{ij} = \dots$

c) nested $\mathcal{O}(n)$-loops quadratic.

⋮

Many algorithms have polynomial runtime

$\Longrightarrow$ Rules suffice

$$ax^n + bx^{n-1} + \dots + z \in \mathcal{O}(x^n)$$

$$\frac{f(x+h)-f(x)}{h} \rightsquigarrow + \mathcal{O}(h^2) \xrightarrow[h \to 0]{}$$

```
for (i=0; i<n; i++)
  for (j=i; j<n; j++)
  {
     O(A(n)) = 1
  }
```

$O(n^2)$

$$\sum_{i=0}^{n} \sum_{i}^{n} 1$$

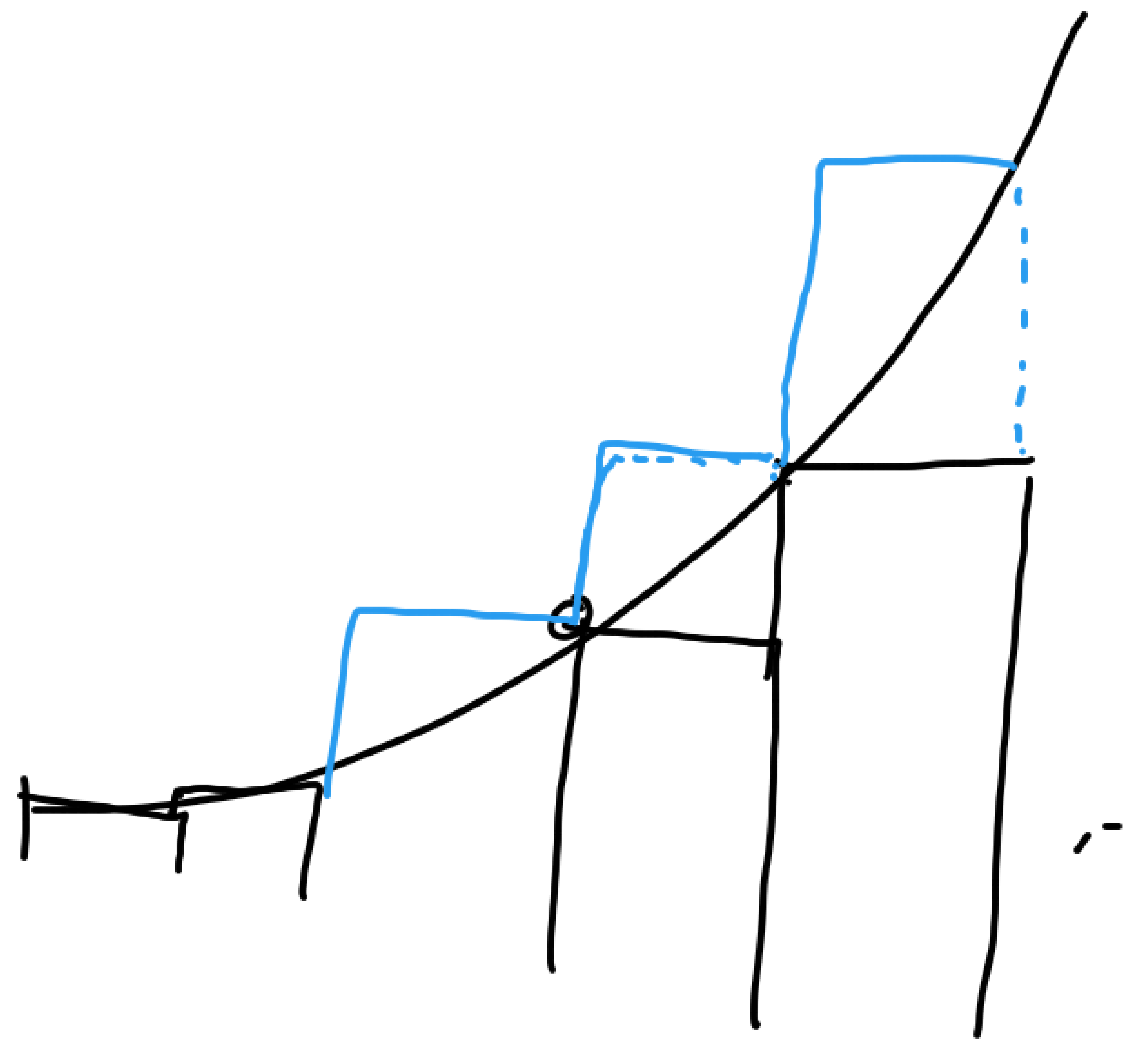
$$= \sum_{0}^{n} \boxed{\sum_{0} i}$$

$1+2+3+4+5$

$O(n)$

$O(n^2)$

$$\sum_{i=0}^{n} i = \frac{n(n-1)}{2} = \frac{1}{2}n^2 + \dots \in O(n^2)$$



a = ['a', 'b', 'c']

a[1]